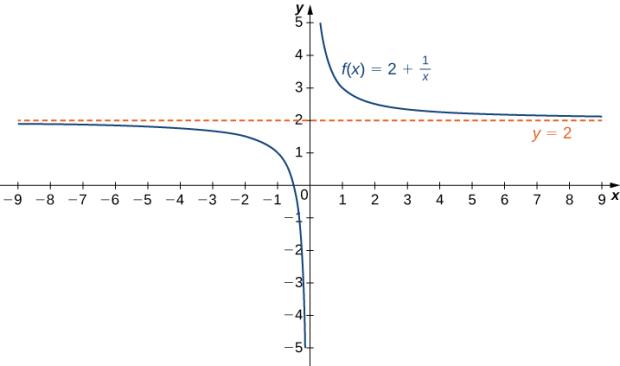
# Section 4.6: Limits at Infinity and Asymptotes

## Limits at Infinity

In this section, we focus on the behavior of a function at the extreme values of and look at horizontal asymptotes.

### Limits at Infinity and Horizontal Asymptotes

Recall that means becomes arbitrarily close to as gets closer to . We extend this idea to limits at infinity. For example, in the graph below, as gets larger (moving to the right in the graph), the values of get closer to . We say the limit as approaches of is and write .



Similarly, as gets smaller (moving to the left in the graph), the values of get closer to . We say the limit as approaches of is and write .

If the values of become arbitrarily close to as becomes sufficiently large, the function has a limit at infinity, written

.

If the values of become arbitrarily close to for as becomes sufficiently large, the function has a limit at negative infinity, written

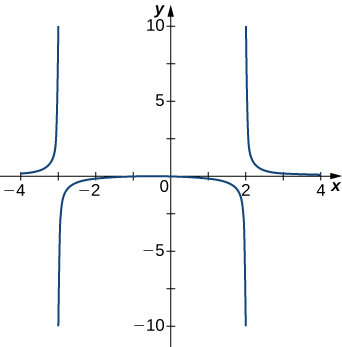
.

If or , the line is a **horizontal asymptote** of .

Media: Watch these [video](https://youtu.be/Z5R8KZJ73UI)1 and [video2](https://youtu.be/k-O6lUN6YG0) examples on limits at infinity and horizontal asymptotes.

Examples:

1. For the graph below, identify where the vertical and horizontal asymptotes are located.

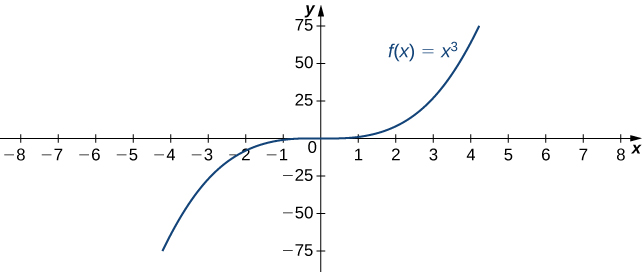


1. For each of the following functions , evaluate and . Determine the horizontal asymptote(s) for .

### Infinite Limits at Infinity

Sometimes the values of a function become arbitrarily large as (or as ). In this case, we write (or ). If the values of a function are negative but become arbitrarily large in magnitude as (or as ), we write (or ).

For example, in the graph below of the function , as , the values of become arbitrarily large. Therefore, .



As , the values of are negative but arbitrarily large. Therefore,.

A function has an **infinite limit at infinity** and write

,

if becomes arbitrarily large for sufficiently large.

Aa function has a negative infinite limit at infinity and write

,

if and becomes arbitrarily large for sufficiently large.

Similarly, we can define infinite limits as .

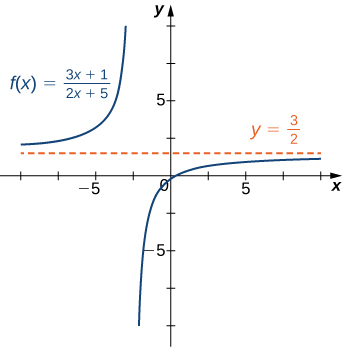
Media: Watch this [video](https://youtu.be/WQNi7hre8u0) example on infinite limits at infinity.

Example: Evaluate .

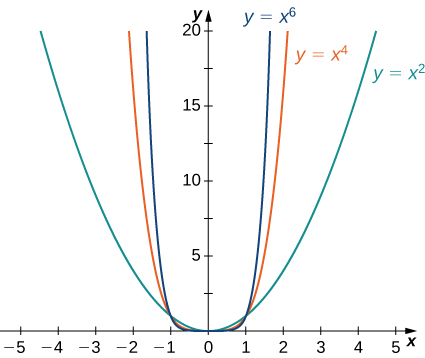
## End Behavior

The behavior of a function as is called the function’s **end behavior**. At each of the function’s ends, the function could exhibit one of the following types of behavior:

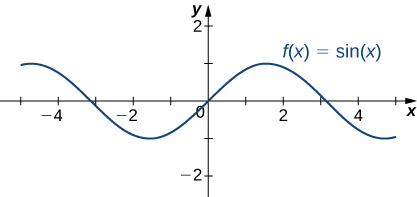
1. The function approaches a horizontal asymptote . Many rational functions have a horizontal asymptote.



1. The function or . Many power functions have this type of behavior.



1. The function does not approach a finite limit, nor does it approach or . In this case, the function may have some oscillatory behavior. Trigonometric functions, like sine and cosine, typically have this type of behavior.



Media: Watch this [video](https://youtu.be/WKwc0sxLRTs) example on asymptotes.

Media: Watch this [video](https://youtu.be/JBOWdxO9iCY?t=104) example on sketching functions with given asymptotes.

Examples

1. Find the horizontal and vertical asymptotes for the function .
2. Construct a function that has the given asymptotes: